Water Bottle Rocket Design Contest

Calculation Exercises

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Middle/High School Division

9th – 12th Grade

**Question 1 Equations:**

**CIRCLE**

|  |  |  |
| --- | --- | --- |
| C = Circumference | π = 22/7 or 3.14, pi | r = Radius = ½ D |
| D = Diameter = (r+r) = 2r | A = Area |  |



**TRIANGLE**

|  |  |
| --- | --- |
| P = Perimeter  a = side length  b = side length  c = side length |  |

 

|  |  |
| --- | --- |
|  | A = Area  b = base of the triangle  h = height of the triangle |

**RECTANGLE/SQUARE/PARALLELOGRAM**



|  |  |  |
| --- | --- | --- |
| P = Perimeter  l = length  w = width  **Rectangle/Square/ Parallelogram** | A = Area  l = length  w = width  **Rectangle/Square** | A = Area  l = length  h = height  **Parallelogram** |

**TRAPEZOID**



|  |  |
| --- | --- |
| P = Perimeter  a = top side length  b = bottom side length  c = left side length  d = right side length | A = Area  h = height  a = top side length  b = bottom side length |

**ELLIPSE**



|  |  |
| --- | --- |
| P = Perimeter  a = major axis  b = minor axis | A = Area  a = major axis  b = minor axis |

1. Skywalker's class constructed ten water bottle rockets.
2. Calculate the circumference of the two-liter bottle nozzle.

Given information: the diameter of a two-liter bottle nozzle 𝐷𝑁𝑜𝑧𝑧𝑙𝑒 = 22.225 𝑚𝑚

1. Determine the ratio of the circumference of the two-liter bottle to the diameter.

Given Information: 𝐷𝑁𝑜𝑧𝑧𝑙𝑒 = 22.225 𝑚𝑚,

1. Determine the area of the two-liter bottle nozzle.
2. Determine the area and perimeter of each fin from all ten water bottle rockets in the chart below. Complete the chart below

Table 1.1: Ten Water Bottle

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Rocket #** | **Fin Shape** | **Base (cm)** | **Height (cm)** | **Major Axis (cm)** | **Minor Axis (cm)** | **Perimeter (cm)** | **Area (cm2)** |
| 1 | Right Triangle | 6 | 10 | - | - |  |  |
| 2 | Right Triangle | 6 | 8 | - | - |  |  |
| 3 | Rectangle | 6 | 10 | - | - |  |  |
| 4 | Rectangle | 6 | 8 | - | - |  |  |
| 5 | Right Trapezoid | 10 | 3 | - | - |  |  |
| 6 | Right Trapezoid | 10 | 4 | - | - |  |  |
| 7 | Trapezoid | 12 | 3 | - | - |  |  |
| 8 | Trapezoid | 12 | 4 | - | - |  |  |
| 9 | Ellipse | - | - | 10 | 6 |  |  |
| 10 | Ellipse | - | - | 10 | 4 |  |  |

1. Calculate the maximum height of the projectile motion of the water bottle rocket.

Use this online calculator to determine the maximum height. You will need velocity (m/s), launch angle, initial height (m). Check your units!

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Rocket #** | **Fin Shape** | **Velocity (m/s)** | **Launch Angle** | **Initial Rocket Height Ho (cm)** | **Maximum Height HMax (m)** |
| 1 | Right Triangle | 6 | 90° | 76 |  |
| 2 | Rectangle | 9 | 90° | 76 |  |
| 3 | Right Trapezoid | 10 | 90° | 76 |  |
| 4 | Trapezoid | 12 | 90° | 76 |  |
| 5 | Ellipse | 17 | 90° | 76 |  |

1. A water bottle rocket is launched straight up from the top of a 24 ft tall building with an initial speed of 92 ft/s. The function ℎ(𝑡) can model the water bottle rocket's height as a function of time ℎ(𝑡) = – 16𝑡2 + 92𝑡 + 24. How long will it take for the water bottle rocket to hit the ground if the height was equal to 95 ft?

h is the height

t is the hangtime in seconds

v is the velocity in ft/s

Use the quadratic equation to calculate the time. Use a graphing calculator to verify your results. Calculate the hangtime of a water bottle rocket using the function. (Hint: ℎ(𝑡) = 0 is when the water bottle rocket hits the ground).

**Question 4 & 5 Example:** A water bottle rocket is launched at an angle of 𝜃 = 90° above the horizontal. Initial time t = 0 and initial velocity of v0=36 ft/s.

Assuming that the water bottle rocket launching was done at ground level, there is no air resistance, and the line joining the landing and impact points is horizontal.

Use Figure 1 to answer the questions below.

* Find the water bottle rocket maximum height attained in after launch in feet.
* Find the water bottle rocket hangtime in seconds.

When vertical velocity = 0

Vertical velocity is:

Gravity is:

[Click to see an example problem.](http://www.phengkimving.com/calc_of_one_real_var/06_the_trig_func_and_their_inv/06_01_the_trig_func/06_01_06_the_proj_motion.htm#fig_02_01)

Chart, line chart

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Example A: Determine the maximum height of the water bottle rocket.

Equation 1:

Equation 2:

Step 1 – Set Equation 2 equal to 0.

Equation 3:

Step 2 – Substitute Equation 3 into Equation 1.

Equation 4:

Step 3 – Substitute the given values into Equation 4 and solve for max height.

Example B: Determine the maximum hangtime of the water bottle rocket.

Equation 1:

Step 1 – Substitute zero into Equation 1 and solve for t.

Step 2 – Substitute in given values.

1. A water bottle rocket is launched at an angle of 𝜃 = 90° above the horizontal. Initial time t = 0 and initial velocity of v0=92 ft/s.

Assuming that the water bottle rocket launching was done at ground level, there is no air resistance, and the line joining the landing and impact points is horizontal.

Use the figure below to answer the questions

* Find the water bottle rocket maximum height attained in after launch in feet.
* Find the water bottle rocket hangtime in seconds.

1. A water bottle rocket is launched at an angle of 𝜃 = 90° above the horizontal. Initial time t = 0 and initial velocity of v0=28m/s.

Assuming that the water bottle rocket launching was done at ground level, there is no air resistance, and the line joining the landing and impact points is horizontal.

Use the figure below to answer the questions

* Find the water bottle rocket maximum height attained in after launch in feet.
* Find the water bottle rocket hangtime in seconds.

For questions 4 & 5:

When vertical velocity = 0

Vertical velocity is:

Gravity is:

Chart, line chart

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**Question 6 Examples:**

The equation for the mass flow rate is the following:

**𝑚̇** is mass flow rate is the amount of water (mass) that flows out of the "rocket nozzle" or throat of the two-liter bottle over a period of time (in a second), and units are 𝑘𝑔/s.

**A** is the nozzle area (hint: it's a circle) of a two-liter bottle.

**Cd** is the discharge coefficient, a dimensionless constant based on nozzle shape (circle) and flow conditions.

is the density of water

∆𝑃𝑁𝑜𝑧𝑧𝑙𝑒 is the pressure drop across the "rocket nozzle" or throat of the two-liter bottle.

𝑃𝑠𝑢𝑝𝑝𝑙𝑦 is the supply pressure, which ALL water bottle rockets are launched with the following pressure:

Below are unit conversation for pressure are the following:

𝑃𝑎𝑡𝑚 is the atmospheric pressure a constant, which is the following value:

Below are unit conversations for mass flow rate are the following:

|  |  |
| --- | --- |
|  |  |
|  |  |

**Example**: What is the mass flow rate for a water bottle rocket with a nozzle diameter of 0.84 inches with a supply pressure of 100 psi, and atmospheric pressure of 760 mmHg?

Please determine the following:

1. Find the area of nozzle of 2-liter bottle in 𝑚2
2. Calculate the pressure drop across the "rocket nozzle" (throat) of the 2-liter bottle in to N/m2
3. Determine what is the mass flow in kg/s
4. *Find the area of nozzle of 2-liter bottle convert to 𝑚2*

Given Information: 𝐷𝑁𝑜𝑧𝑧𝑙𝑒 = 0.84 𝑖𝑛𝑐ℎ𝑒𝑠

Convert the radius to meters from inches

1. Calculate the pressure drop across the "rocket nozzle" (throat) of the 2-liter bottle in to N/m2 Given Information:

|  |  |  |  |
| --- | --- | --- | --- |
| 𝑃𝑆𝑢𝑝𝑝𝑙𝑦=100𝑝𝑠𝑖 | 𝑃𝑎𝑡m=760𝑚𝑚𝐻𝑔 | 14.7𝑝𝑠𝑖=760𝑚𝑚𝐻𝑔 | 14.7𝑝𝑠𝑖=1.013529 × 105𝑃𝑎 |

1. Determine what is the mass flow in kg/s

Given Information: 𝐶𝑑 = 0.98 𝜌𝐻2𝑂 = 998 kg/𝑚3

1. Determine the mass flow rate of the water bottle rocket:

Step 1: Convert the nozzle diameter from centimeters to meters. (1000 mm = 1 cm)

DNozzle= 22.225 mm

Step 2: Determine the area of the nozzle of the two-liter bottle.

Step 4: Determine the pressure drop across the "rocket nozzle" or throat of the two-liter bottle.

Step 5: Convert 𝛥𝑃𝑁𝑜𝑧𝑧𝑙𝑒 from psi to Pa.

Step 6: Calculate the mass flow rate for the bottle.

Demonstrate that the dimensions cancel out to give you mass units per time units.

**Question 7 Example**

The equation for the exit velocity is flow rate is the following:

**𝑉E𝑥𝑖𝑡**is the exit velocity of the water from the water bottle rocket, and units are 𝑚/s.

**𝑚̇** is mass flow rate is the amount of water (mass) that flows out of the "rocket nozzle" or throat of the two-liter bottle over a period of time (in a second), and units are 𝑘𝑔/s.

**A** is the nozzle area (hint: it's a circle) of a two-liter bottle.

is the density of water

**Example**: What is the exit velocity of the water from the rocket? The answer should be in m/s.

Given Information

|  |  |  |
| --- | --- | --- |
|  |  |  |

1. What is the exit velocity in m/s of the water bottle rocket in problem 6?

Show your work!

1. Calculate the following using the measurements from your SECME team’s water bottle:
2. The diameter of the water bottle rocket nozzle used to construct your rocket convert from centimeters to inches.
3. The area of the water bottle rocket nozzle used to construct your rocket convert from meters to feet.
4. Determine the pressure drop across the water bottle rocket nozzle or throat in 𝑃𝑎, 𝑚𝑚 𝐻𝑔, and 𝑡𝑜𝑟𝑟.
5. Determine the mass rate of the water bottle rocket used to construct your rocket in both 𝑘𝑔/s and 𝑙𝑏𝑓.
6. Determine the exit velocity of the water bottle rocket used to construct your rocket in 𝑚/s to 𝑚𝑝ℎ = 𝑚𝑖𝑙𝑒𝑠/ℎ𝑜𝑢𝑟.

CALCULATION SHEET

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